



Statistical approach of parton distributions: a closer look at the high- x region

Jacques Soffer

*Department of Physics, Temple University, Philadelphia, PA 19122-6082,
USA*

Outline

- **Basic procedure** to construct the statistical polarized parton distributions
- **Essential features** from unpolarized and polarized Deep Inelastic Scattering data
- **Predictions** tested against new data : DIS, Semi-inclusive DIS and several hadronic processes
- **W physics**
- **Conclusions**

Collaboration with [Claude Bourrely](#) and [Franco Buccella](#)

- A Statistical Approach for Polarized Parton Distributions
Euro. Phys. J. [C23](#), 487 (2002)
- Recent Tests for the Statistical Parton Distributions
Mod. Phys. Letters [A18](#), 771 (2003)
- The Statistical Parton Distributions: status and prospects
Euro. Phys. J. [C41](#),327 (2005)
- The extension to the transverse momentum of the statistical parton distributions
Mod. Phys. Letters [A21](#), 143 (2006)
- Strangeness asymmetry of the nucleon in the statistical parton model
Phys. Lett. [B648](#), 39 (2007)
- How is transversity related to helicity for quarks and antiquarks in a proton?
Mod. Phys. Letters [A24](#), 1889 (2009)
- Semiinclusive DIS cross sections and spin asymmetries in the quantum statistical parton distributions approach
(arXiv:1008.5322 (hep-ph))

Our motivation and goal

- Will propose a statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.
- Will incorporate some QCD features

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- Will propose a statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.
- Will incorporate some QCD features
- Will parametrize our PDF in terms of a very few number of physical parameters, at variance with standard polynomial type parametrizations
- Will be able to construct simultaneously unpolarized and polarized PDF: a unique case on the market!
- Will be able to describe physical observables both in DIS and hadronic collisions

Basic procedure

Use a simple description of the PDF, at input scale Q_0^2 , proportional to $[\exp[(x - X_{0p})/\bar{x}] \pm 1]^{-1}$, *plus* sign for quarks and antiquarks, corresponds to a **Fermi-Dirac** distribution and *minus* sign for gluons, corresponds to a **Bose-Einstein** distribution. X_{0p} is a constant which plays the role of the *thermodynamical potential* of the parton p and \bar{x} is the *universal temperature*, which is the same for all partons.

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From the chiral structure of QCD, we have **two important properties**, allowing to relate quark and antiquark distributions and to restrict the gluon distribution:

- Potential of a quark q^h of helicity h is opposite to the potential of the corresponding antiquark \bar{q}^{-h} of helicity $-h$, $X_{0q}^h = -X_{0\bar{q}}^{-h}$.
- Potential of the gluon G is zero, $X_{0G} = 0$.

The polarized PDF at $Q_0^2 = 4\text{GeV}^2$

For light quarks $q = u, d$ of helicity $h = \pm$, we take

$$xq^{(h)}(x, Q_0^2) = \frac{AX_{0q}^h x^b}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1},$$

consequently for antiquarks of helicity $h = \mp$

$$x\bar{q}^{(-h)}(x, Q_0^2) = \frac{\bar{A}(X_{0q}^h)^{-1} x^{2b}}{\exp[(x + X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1}.$$

Note: $q = q^+ + q^-$ and $\Delta q = q^+ - q^-$ (idem for \bar{q}).

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For strange quarks and antiquarks, s and \bar{s} , given our poor knowledge on both unpolarized and polarized distributions, we first took in 2002

$$xs(x, Q_0^2) = x\bar{s}(x, Q_0^2) = \frac{1}{4}[x\bar{u}(x, Q_0^2) + x\bar{d}(x, Q_0^2)]$$

and

$$x\Delta s(x, Q_0^2) = x\Delta\bar{s}(x, Q_0^2) = \frac{1}{3}[x\Delta\bar{d}(x, Q_0^2) - x\Delta\bar{u}(x, Q_0^2)].$$

However given the **strange quark asymmetry**, this was improved in Phys. Lett. **B648**, 39 (2007).

For gluons we use a **Bose-Einstein** expression given by $xG(x, Q_0^2) = \frac{A_G x^{b_G}}{\exp(x/\bar{x}) - 1}$, with a **vanishing potential** and the same temperature \bar{x} . We also need to specify the polarized gluon distribution and we take the particular choice $x\Delta G(x, Q_0^2) = 0$.

Essential features from the DIS data

From well established features of u and d extracted from DIS data, we anticipate some simple relations between the potentials:

- $u(x)$ dominates over $d(x)$, so we should have

$$X_{0u}^+ + X_{0u}^- > X_{0d}^+ + X_{0d}^-$$

- $\Delta u(x) > 0$, therefore $X_{0u}^+ > X_{0u}^-$
- $\Delta d(x) < 0$, therefore $X_{0d}^- > X_{0d}^+$.

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So we expect X_{0u}^+ to be the largest potential and X_{0d}^+ the smallest one. In fact, from our fit we have obtained the following ordering (see below)

$$X_{0u}^+ > X_{0d}^- \sim X_{0u}^- > X_{0d}^+ .$$

This ordering has important consequences for \bar{u} and \bar{d} , namely

- $\bar{d}(x) > \bar{u}(x)$, flavor symmetry breaking expected from **Pauli exclusion principle**. This was already confirmed by the violation of the **Gottfried sum rule** (NMC).
- $\Delta\bar{u}(x) > 0$ and $\Delta\bar{d}(x) < 0$, a **PREDICTION** in agreement with polarized DIS (see below) and will be more precisely checked at RHIC-BNL from W^\pm production.

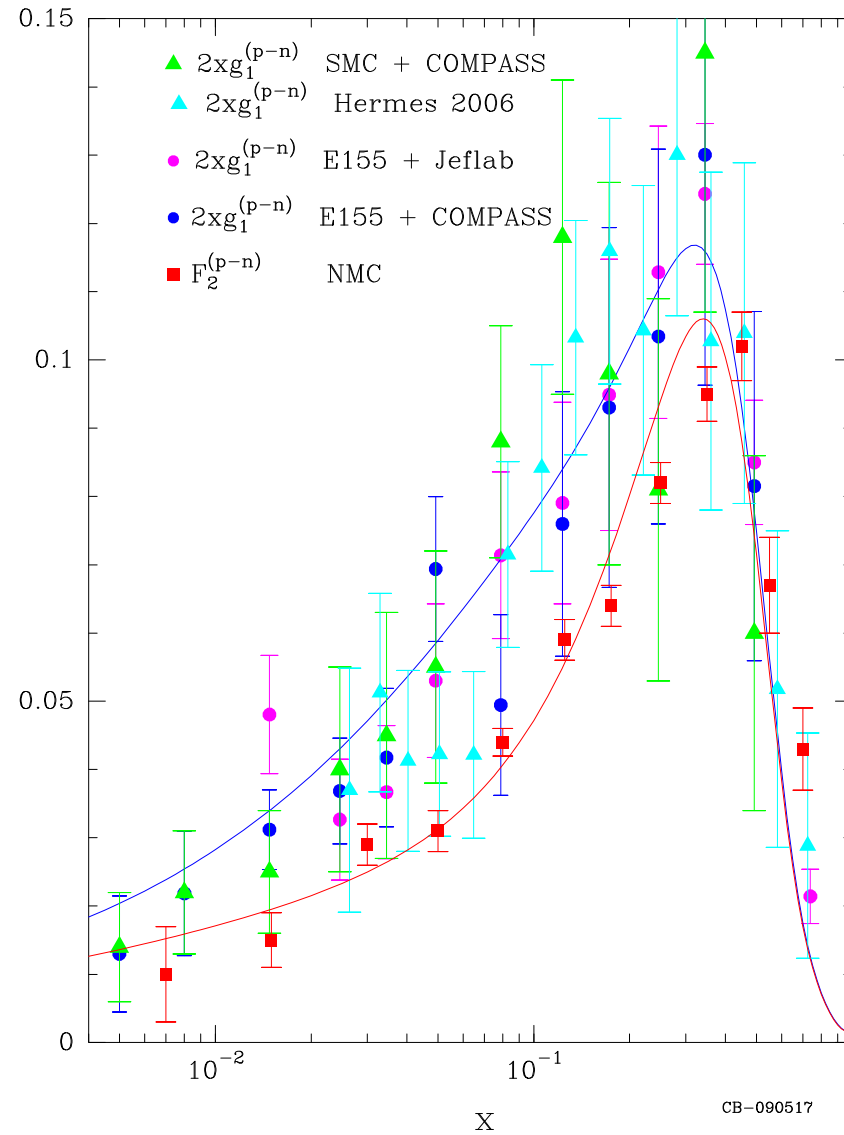
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- Note that since $u^-(x) \sim d^-(x)$, it follows that $\bar{u}^+(x) \sim \bar{d}^+(x)$, so we have

$$\Delta\bar{u}(x) - \Delta\bar{d}(x) \sim \bar{d}(x) - \bar{u}(x) ,$$

i.e. the flavor symmetry breaking is almost the **same** for unpolarized and polarized distributions ($\Delta\bar{u}$ and $\Delta\bar{d}$ contribute to about 10% to the **Bjorken sum rule**).

An interesting observation

u^+ dominates and $u^- \simeq d^-$



Nine free parameters

By performing a NLO QCD evolution of these PDF, we were able to obtain a good description of a large set of very precise data on $F_2^p(x, Q^2)$, $F_2^n(x, Q^2)$, $xF_3^{\nu N}(x, Q^2)$ and $g_1^{p,d,n}(x, Q^2)$, in correspondance with **nine** free parameters with some physical significance:

- * the four potentials X_{0u}^+ , X_{0u}^- , X_{0d}^- , X_{0d}^+ ,
- * the universal temperature \bar{x} ,
- * **and** b , \tilde{b} , b_G , \tilde{A} .

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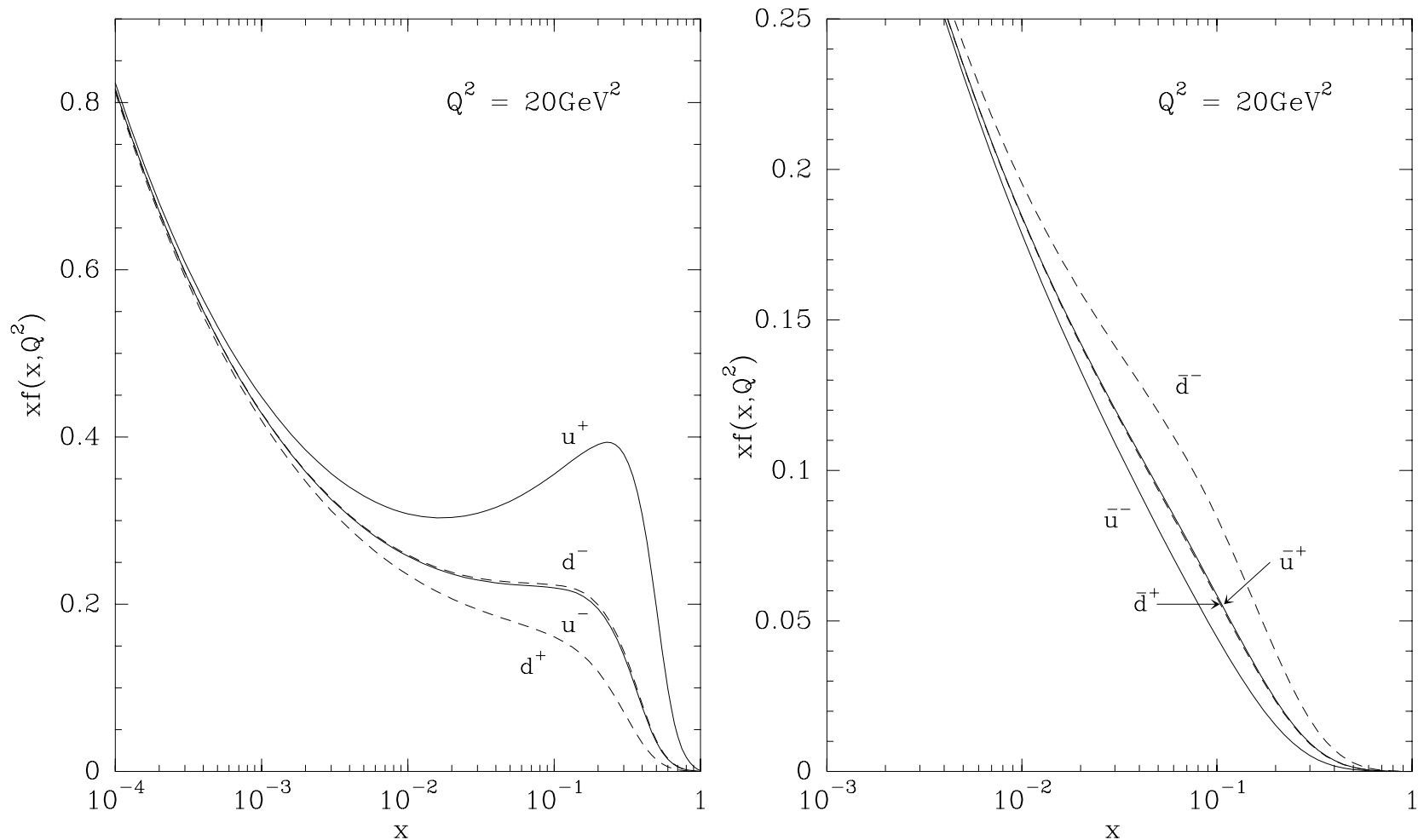
We also have three additional parameters, A , \bar{A} , A_G , which are fixed by 3 normalization conditions .

$$u - \bar{u} = 2, \quad d - \bar{d} = 1$$

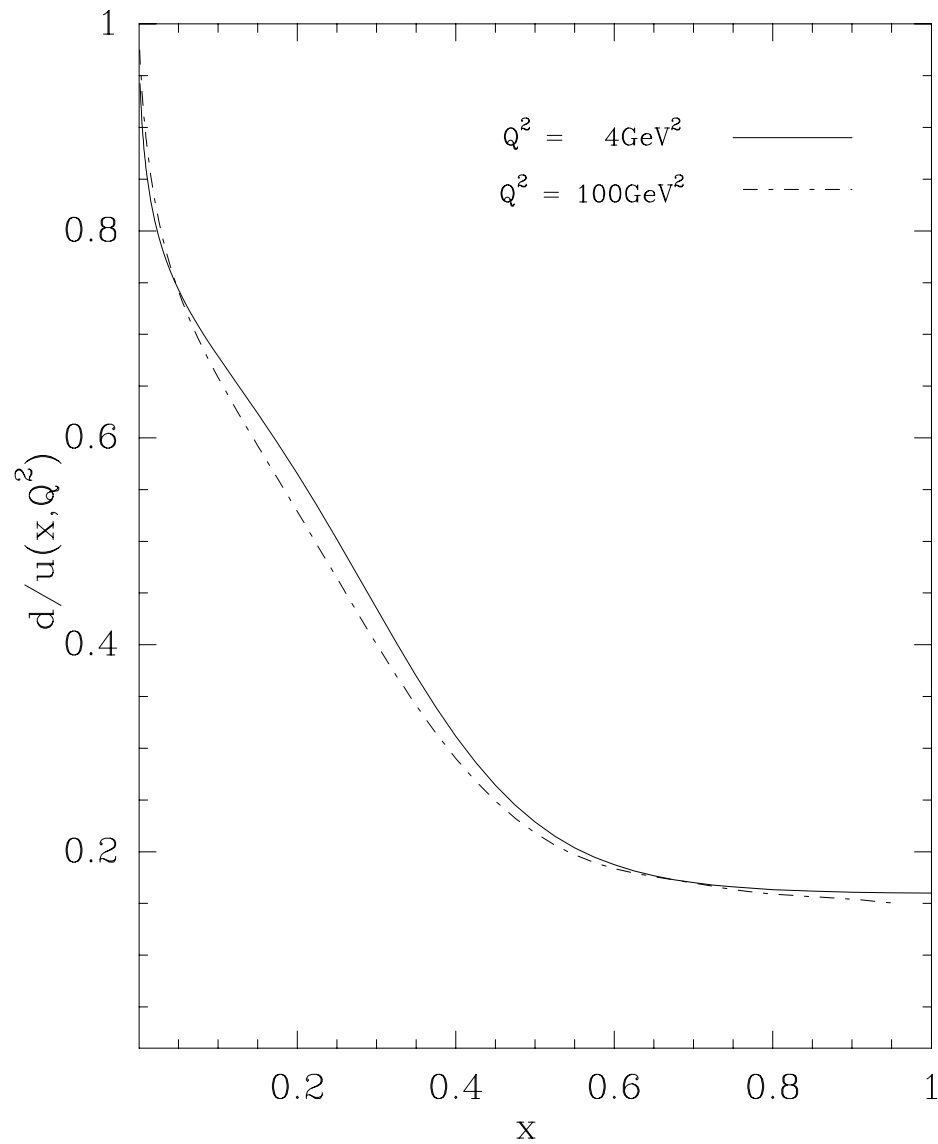
and the momentum sum rule.

Polarized light quarks distributions versus x

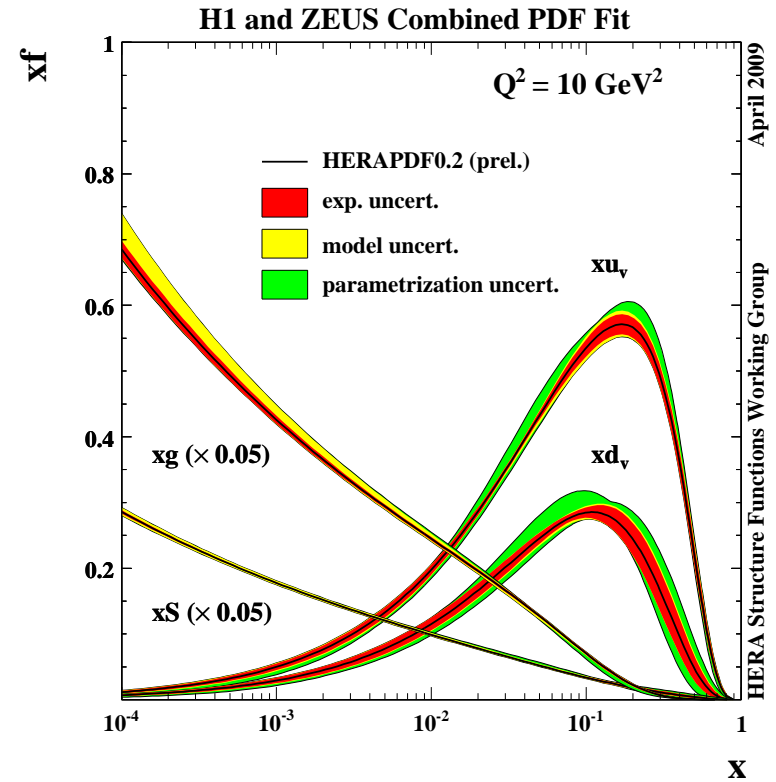
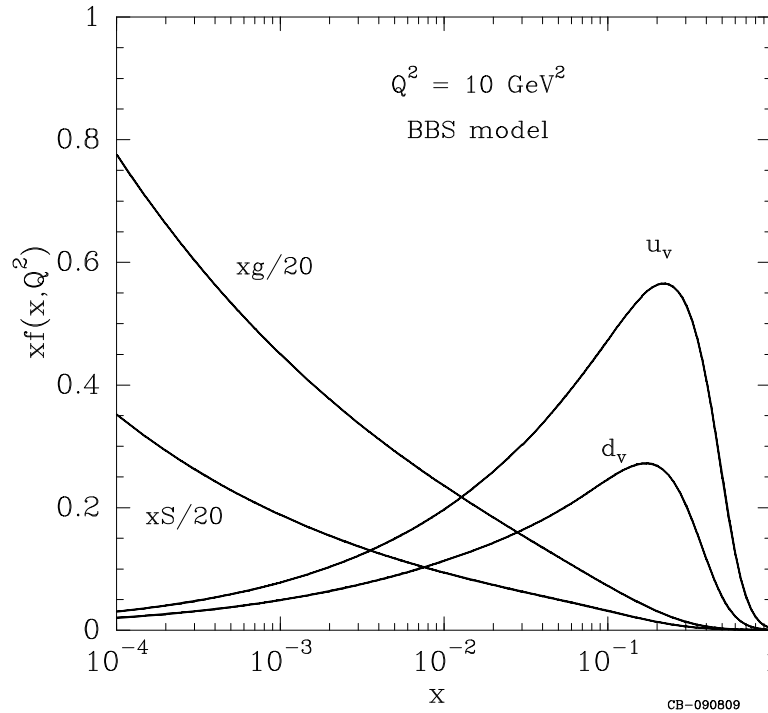
As we could anticipated u^+ , is the largest one, is maximum near $x = 0.3$ and $u^- \simeq d^-$.
Therefore for the antiquarks \bar{d}^- is the largest one and $\bar{u}^+ \simeq \bar{d}^+$
Moreover we find $\Delta\Sigma(Q_0^2) = 0.28$



The d/u ratio versus x



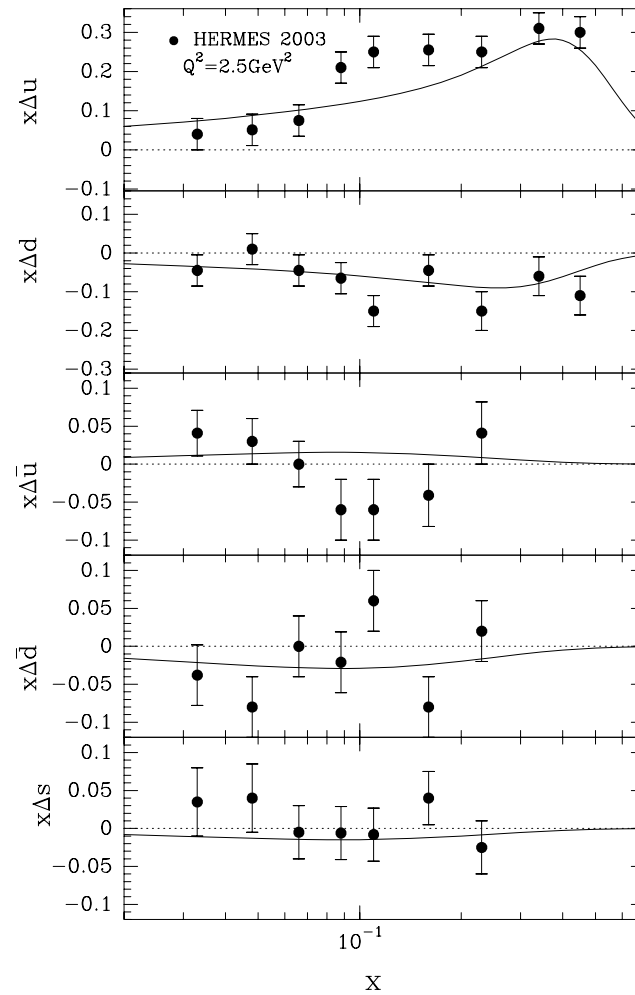
A global view of the unpolarized parton distributions



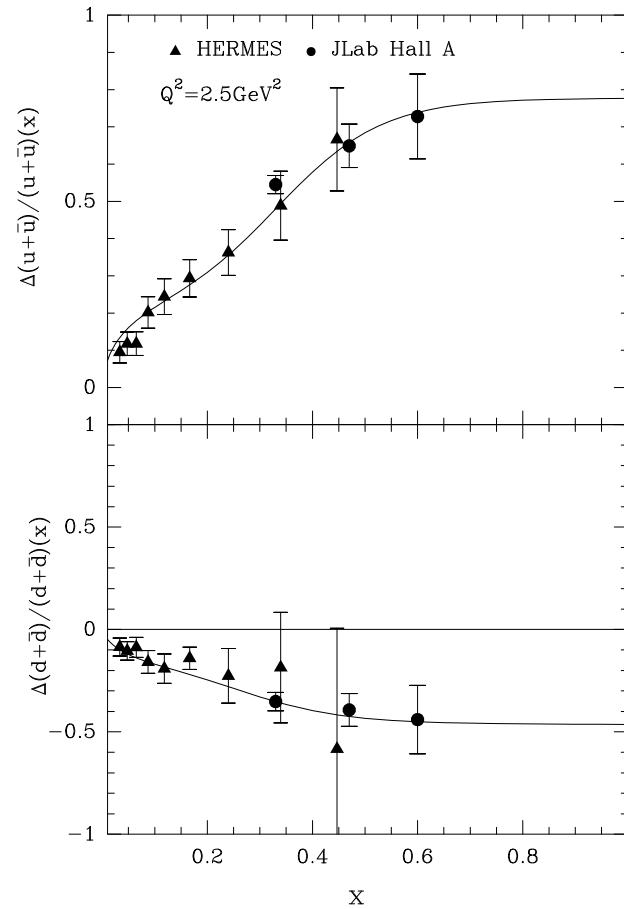
Predictions tested against some data 2002 - 2005

- Deep Inelastic Scattering
- Hadronic Collisions

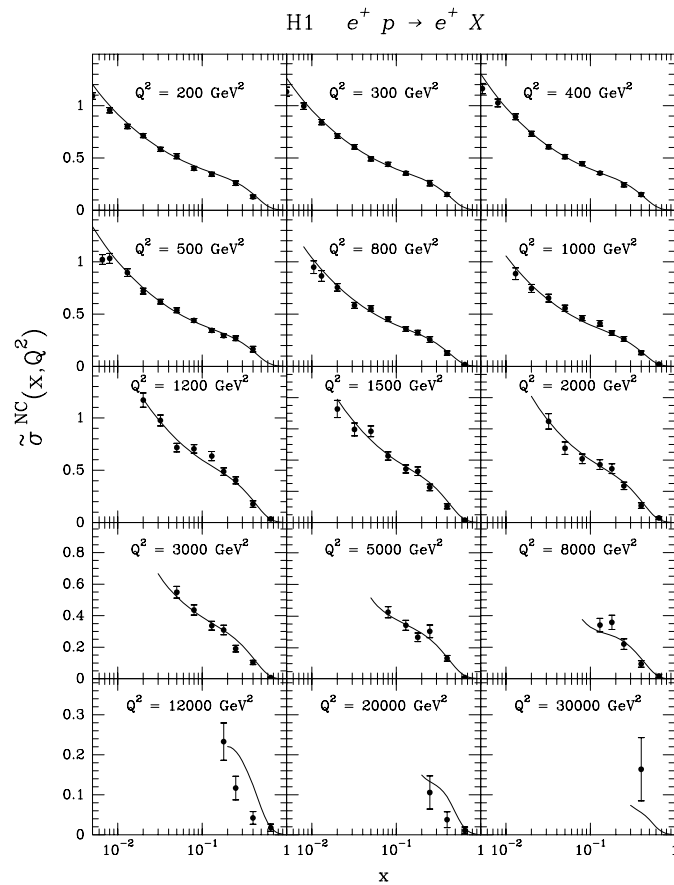
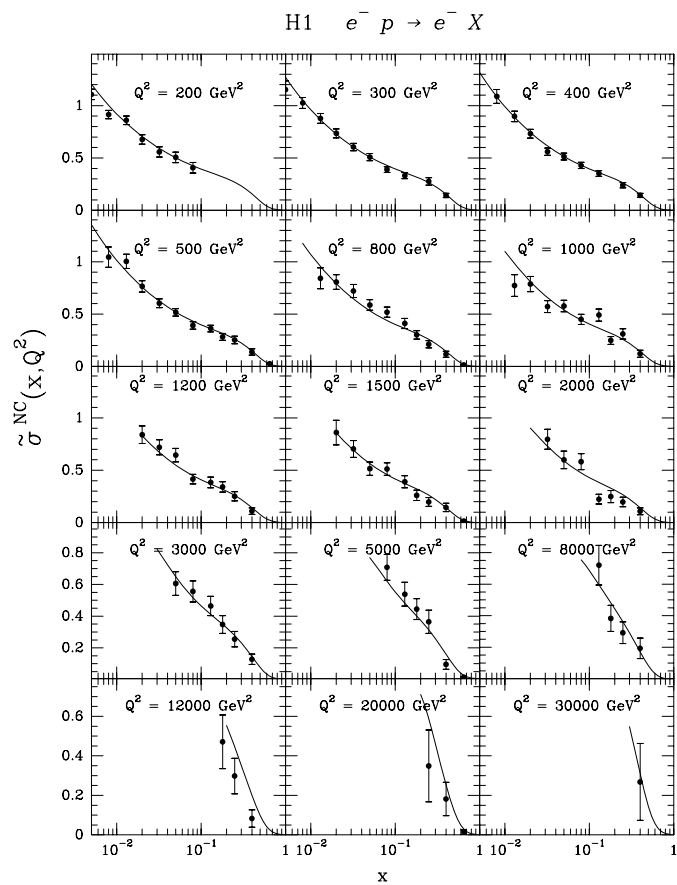
Helicity distributions versus x at DESY and JLab (2004)



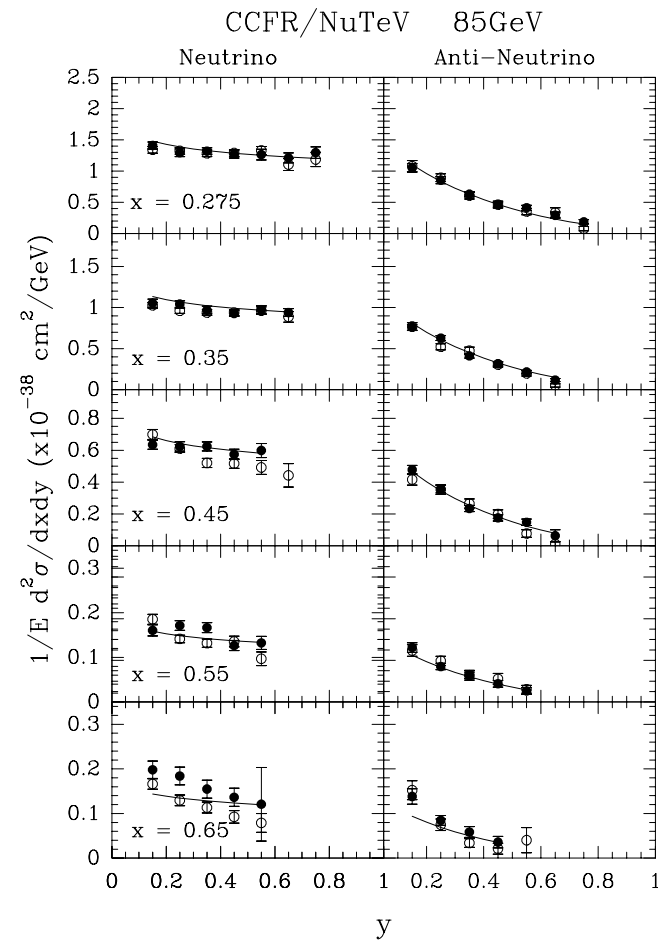
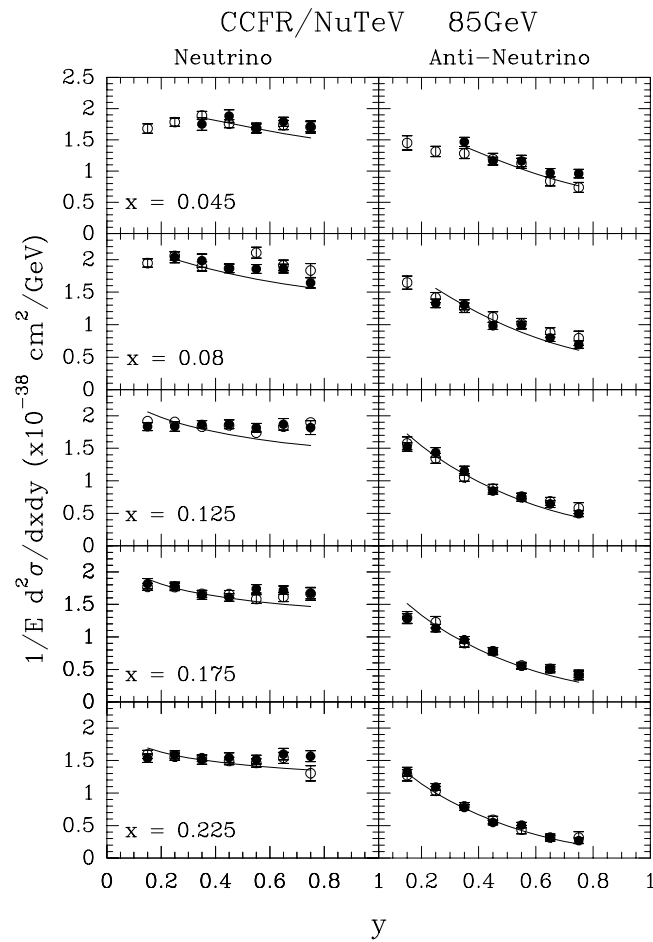
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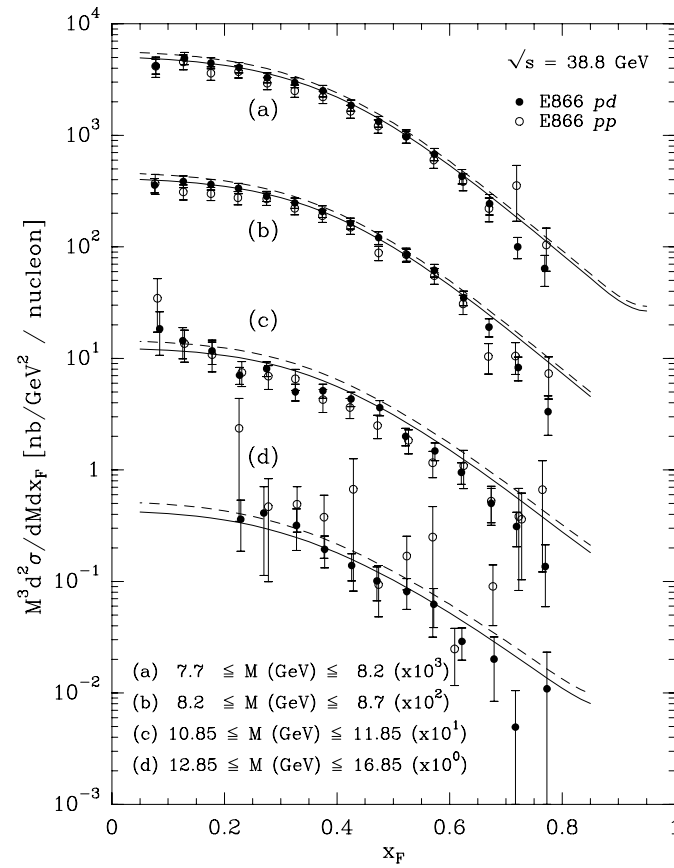
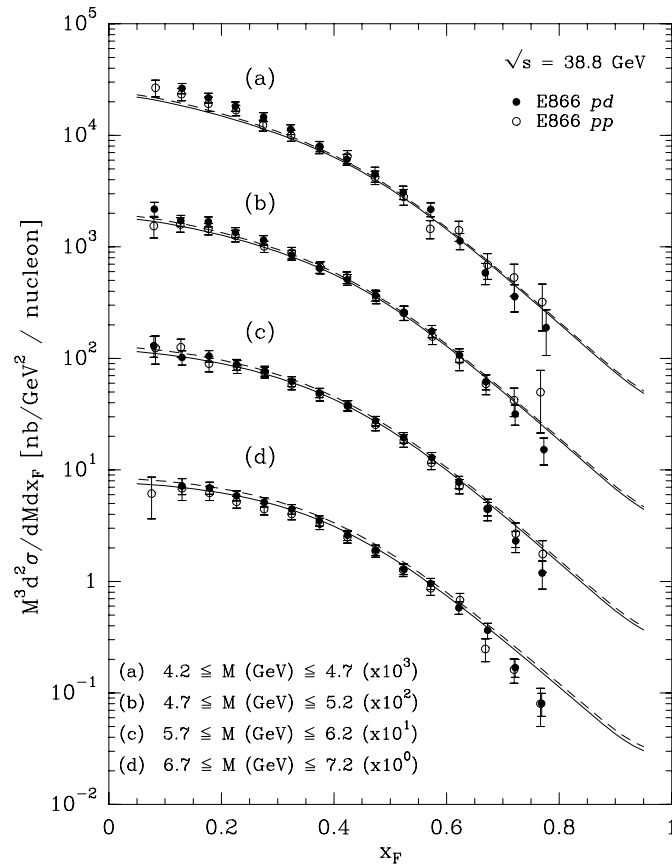
Neutral current in $e^\pm p$ collisions (H1) (2003)



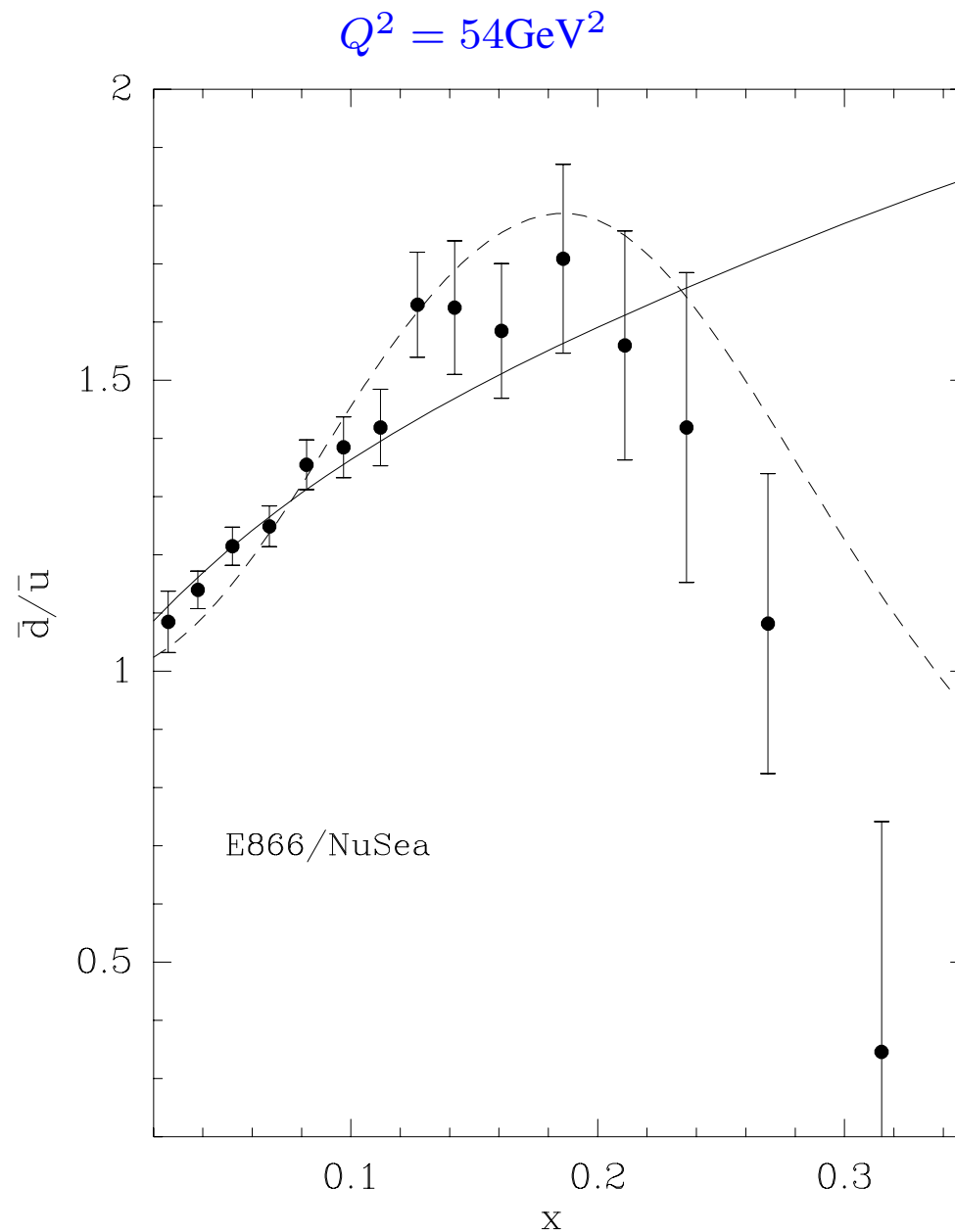
Charged current neutrino cross sections at FNAL (2004)



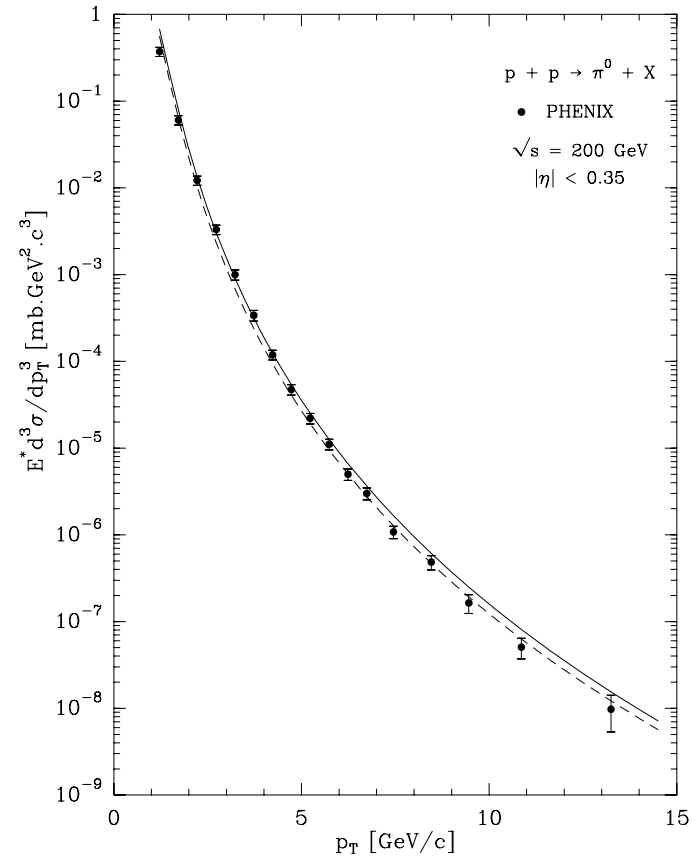
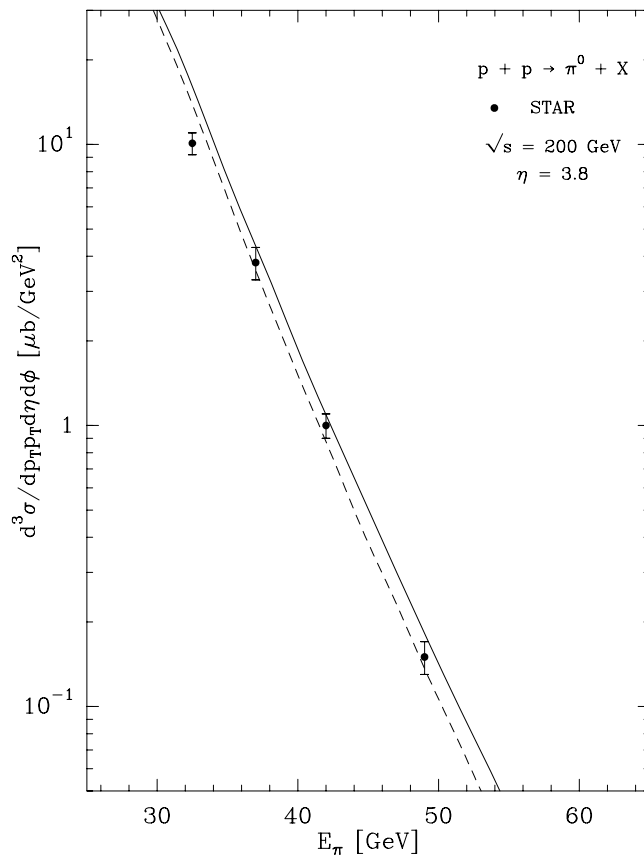
Drell-Yan processes at FNAL (2003)



The important issue of \bar{d}/\bar{u} at large x

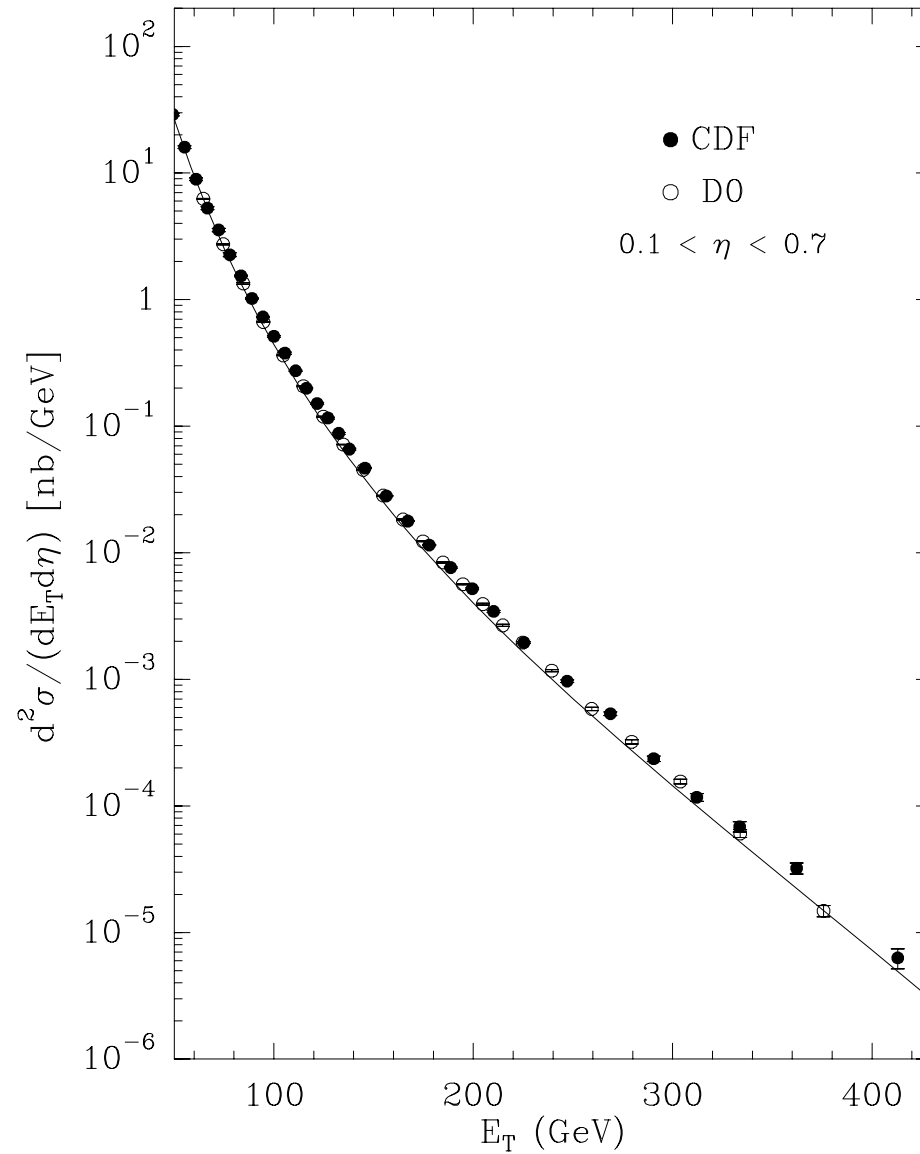


Inclusive π^0 production in pp collisions at RHIC (2003)



Mid-rapidity and central region

Single-jet production in $\bar{p}p$ collisions at FNAL



Predictions tested against some very recent data

- Unpolarized Deep Inelastic Scattering

- Gluon

- * The structure function F_L is a direct sensitivity to the gluon: $F_L = 0$ in quark-parton model, but $F_L \neq 0$ in NLO pQCD

- Valence light quarks

- From $\gamma - Z$ interference in neutral current $e^\pm p$ collisions

- Strange quark and antiquark

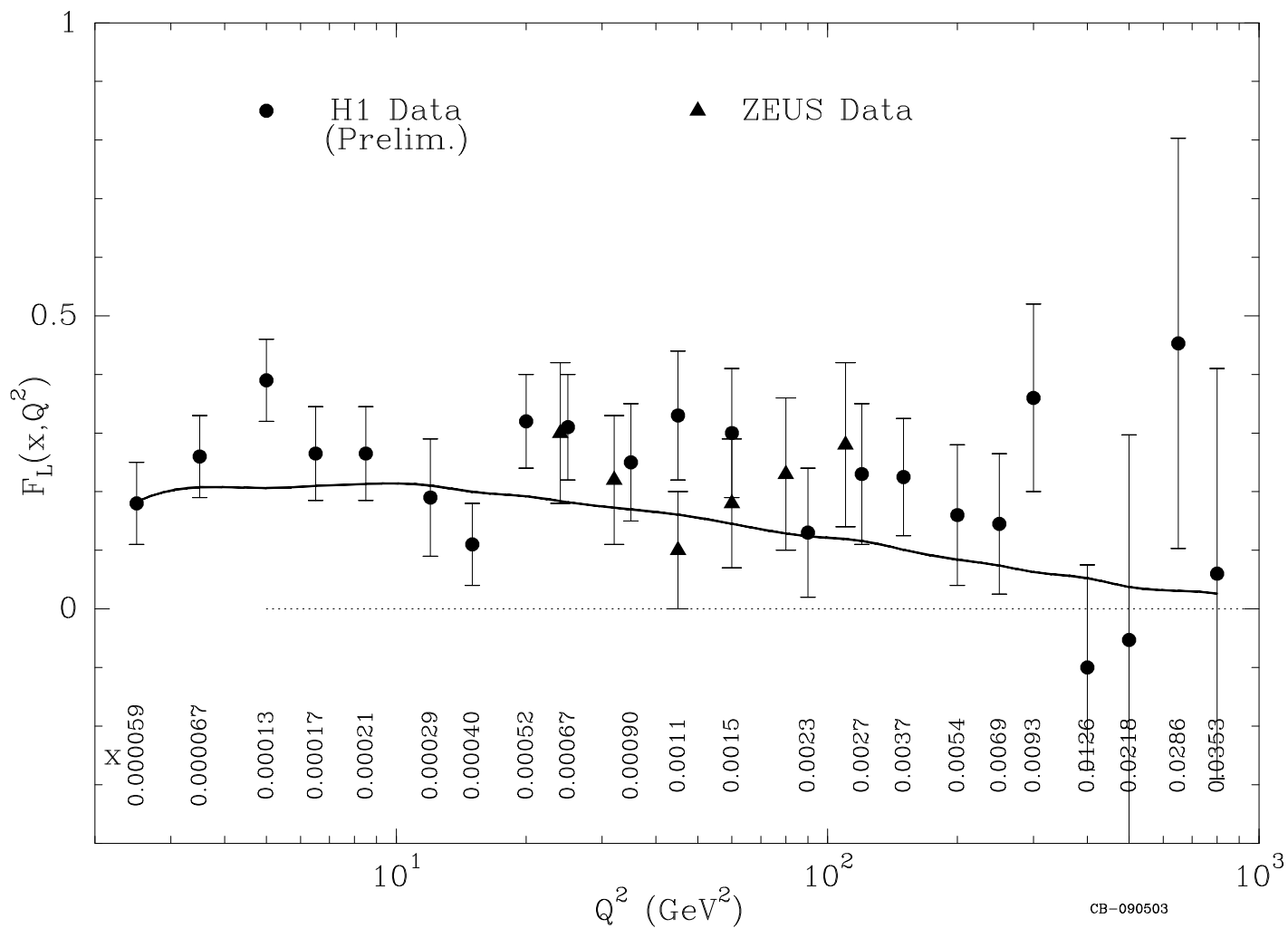
- First determined from NuTeV and tested against Semi-inclusive DIS from Hermes

- Polarized Deep Inelastic Scattering

- * Polarized valence light quarks from Semi-inclusive DIS on Deuterium

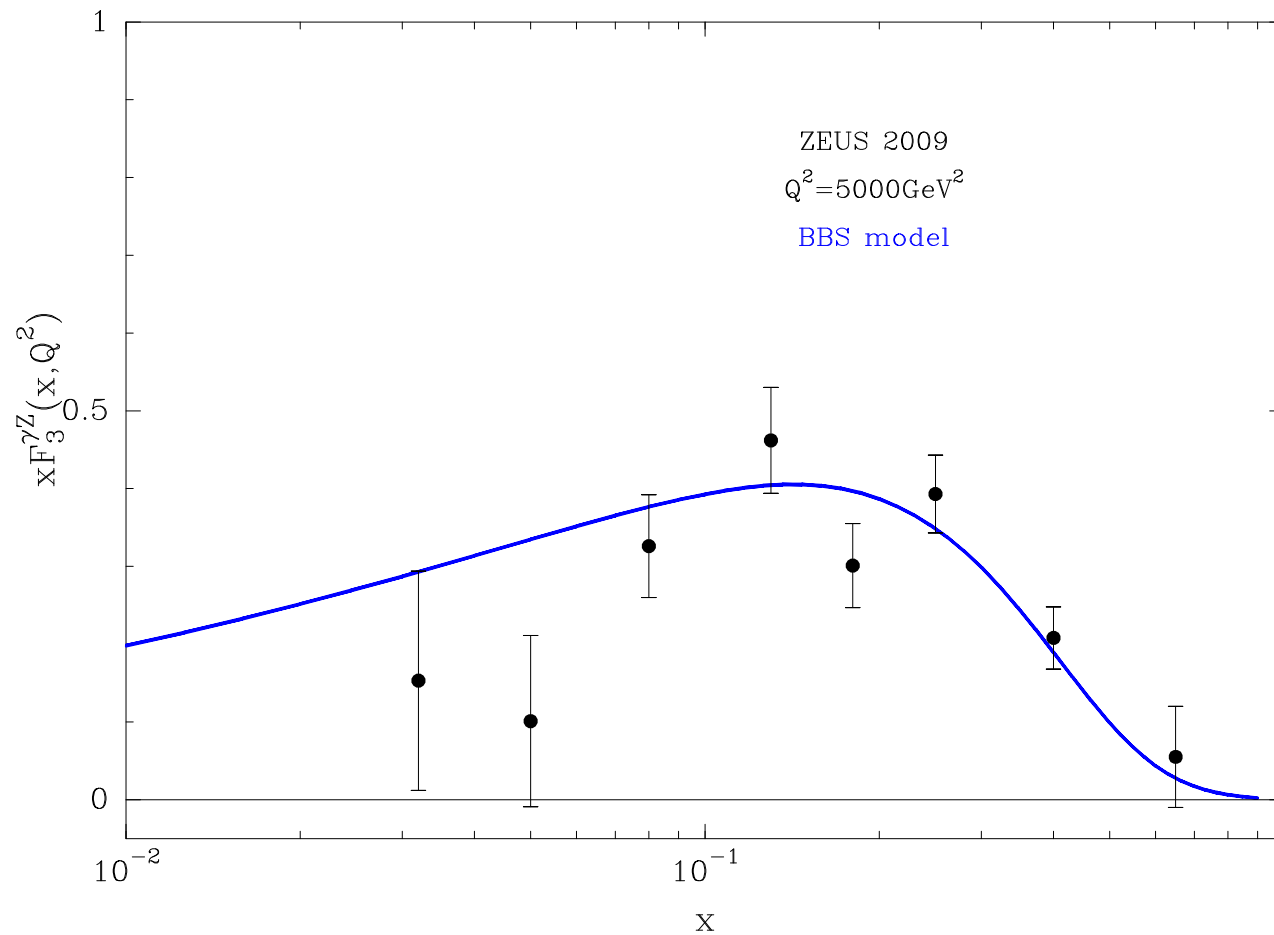
- * Non-symmetric polarized sea quarks

The longitudinal structure function F_L



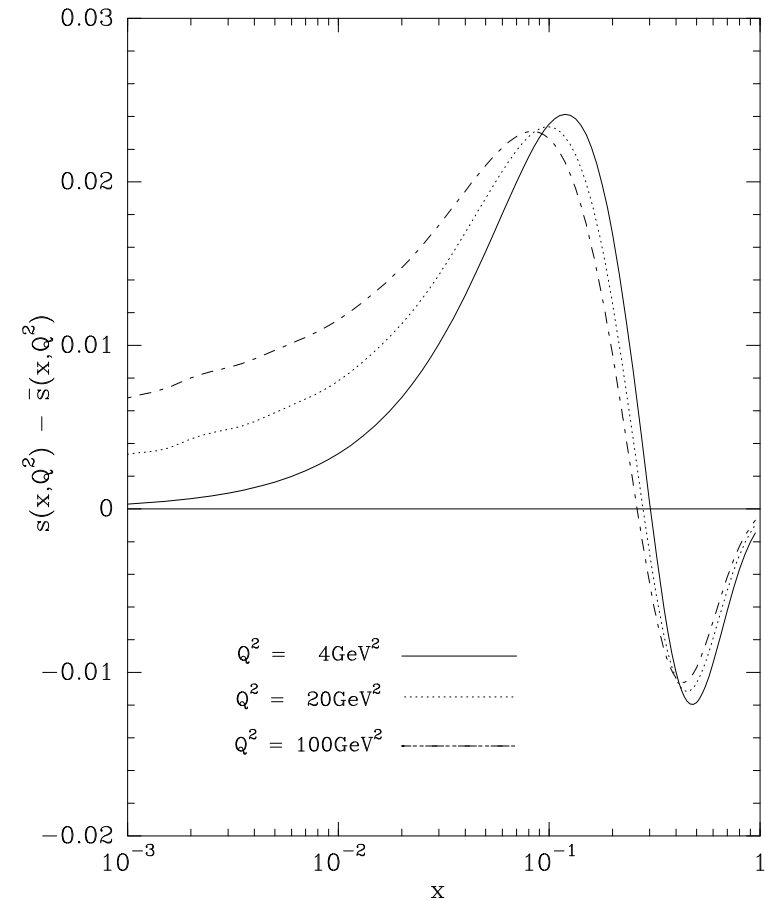
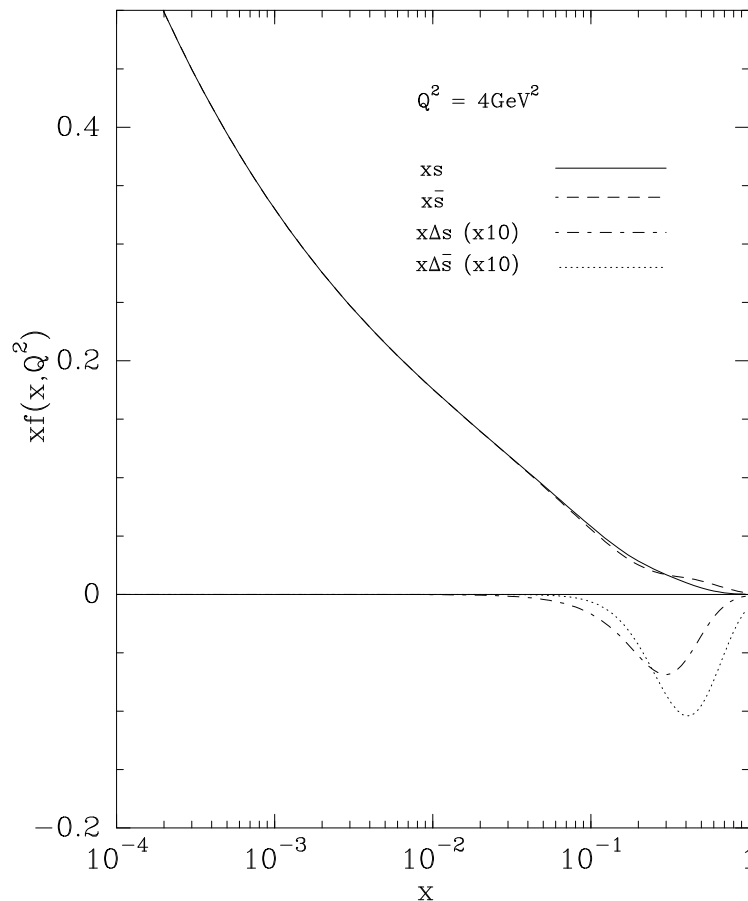
The structure function $F_3^{\gamma Z}$

Interference term which can be isolated in neutral current $e^\pm p$ collisions at high Q^2
We have to a good approximation $x F_3^{\gamma Z} = \frac{x}{3}(2u_v + d_v)$



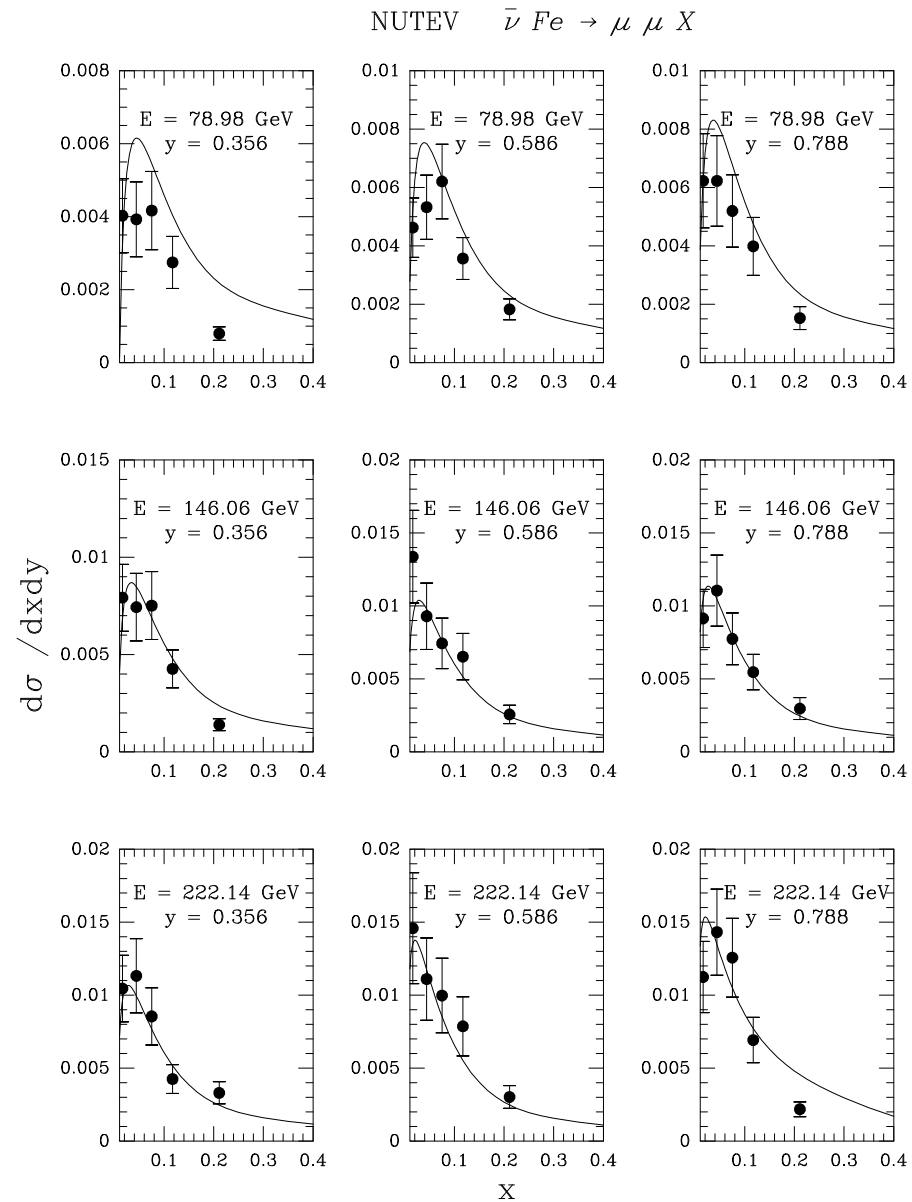
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The strange quark and antiquark distributions

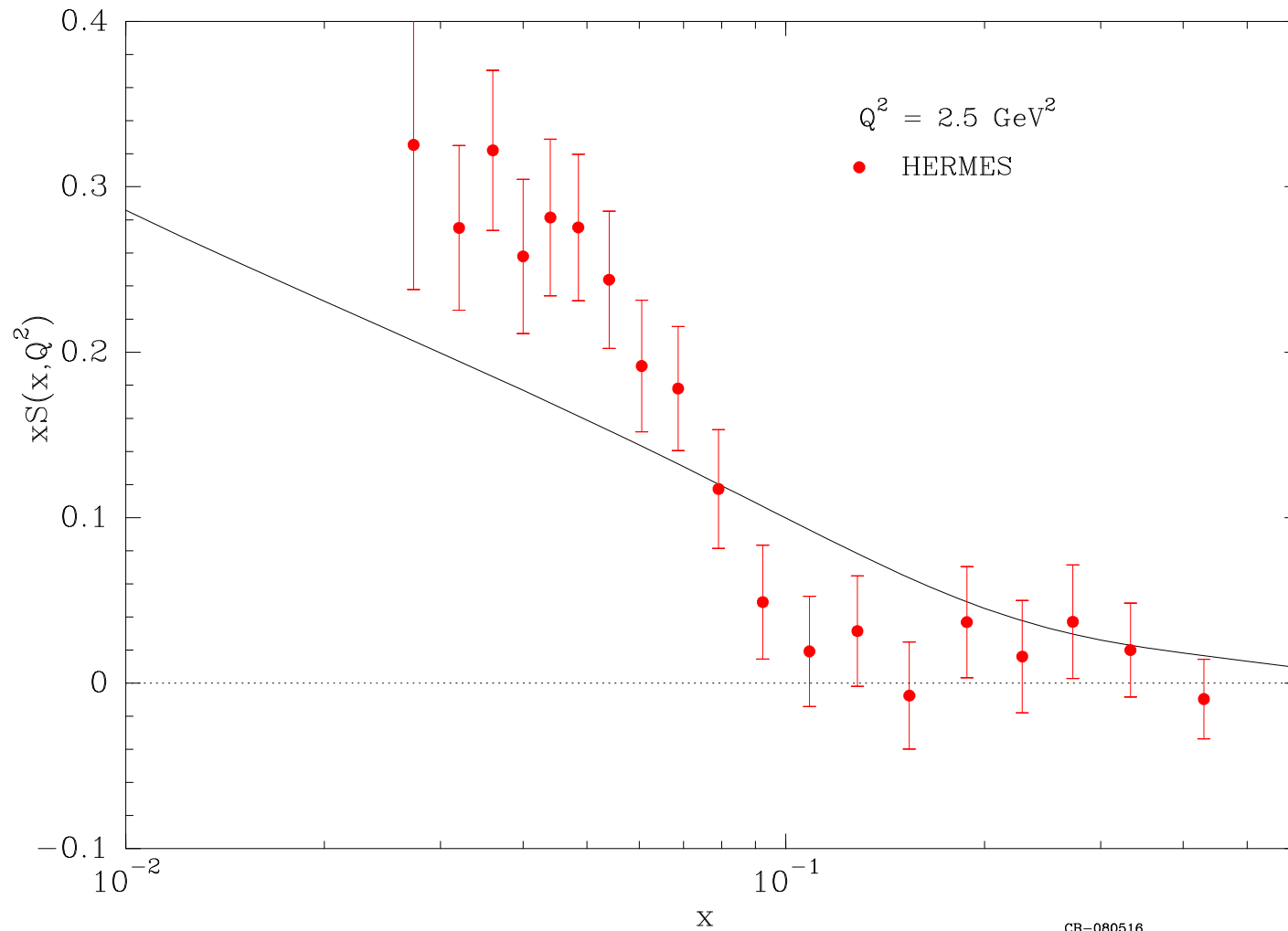


This requires four new parameters X_{0s}^{\pm} , b_s , \tilde{A}_s to fit the CCFR and NuTeV neutrino data for dimuon production

The antineutrino NuTeV data

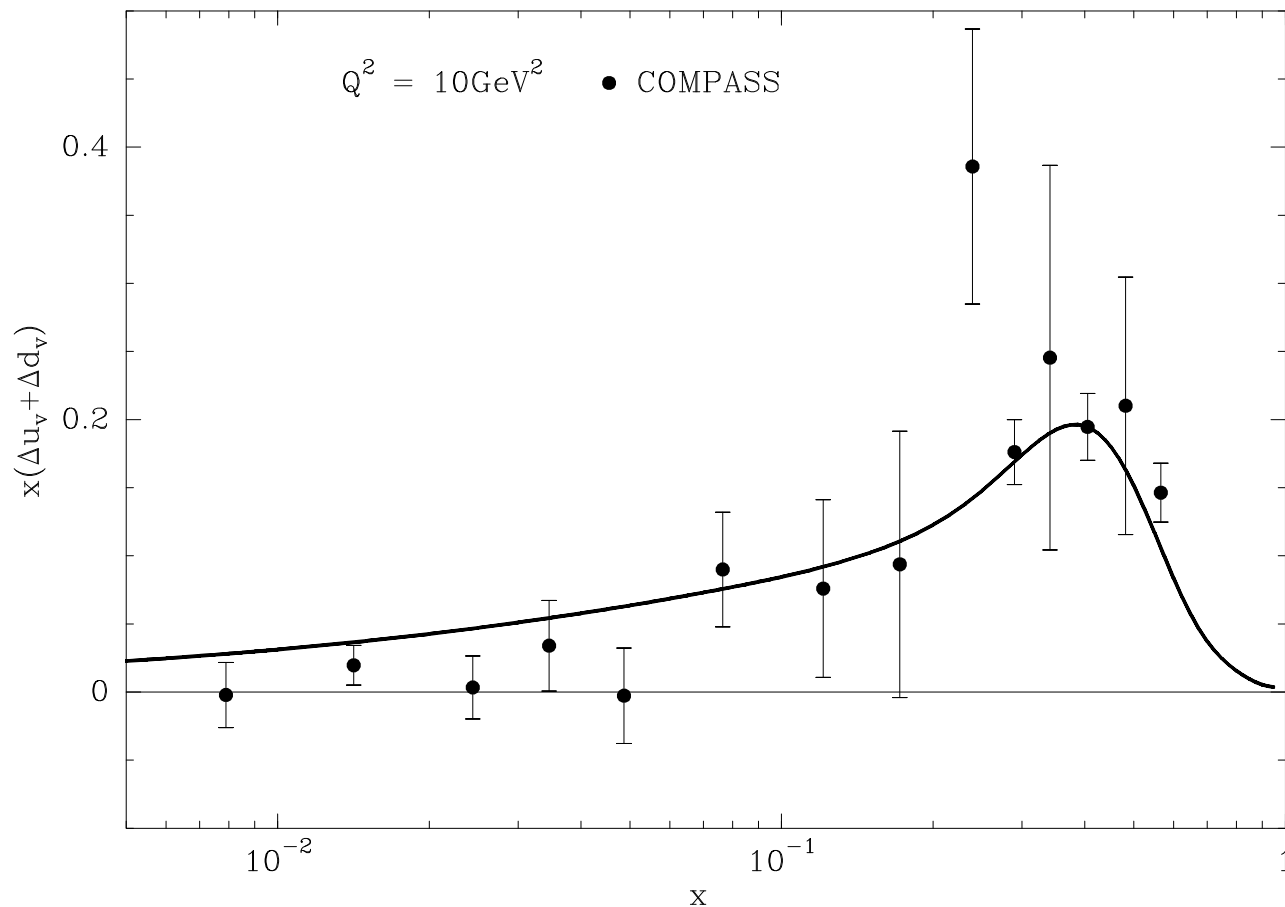


The $xS(x) = xs(x) + x\bar{s}(x)$ distribution from Hermes



The valence quark helicity distributions versus x

From semi-inclusive DIS $\mu p \rightarrow \mu h^\pm X$ can determine the valence quark helicity distributions
Combined with g_1^d it leads to $\Delta\bar{u} + \Delta\bar{d} = 0.0 \pm 0.04 \pm 0.03$, *i.e.* non-symmetric polarized sea



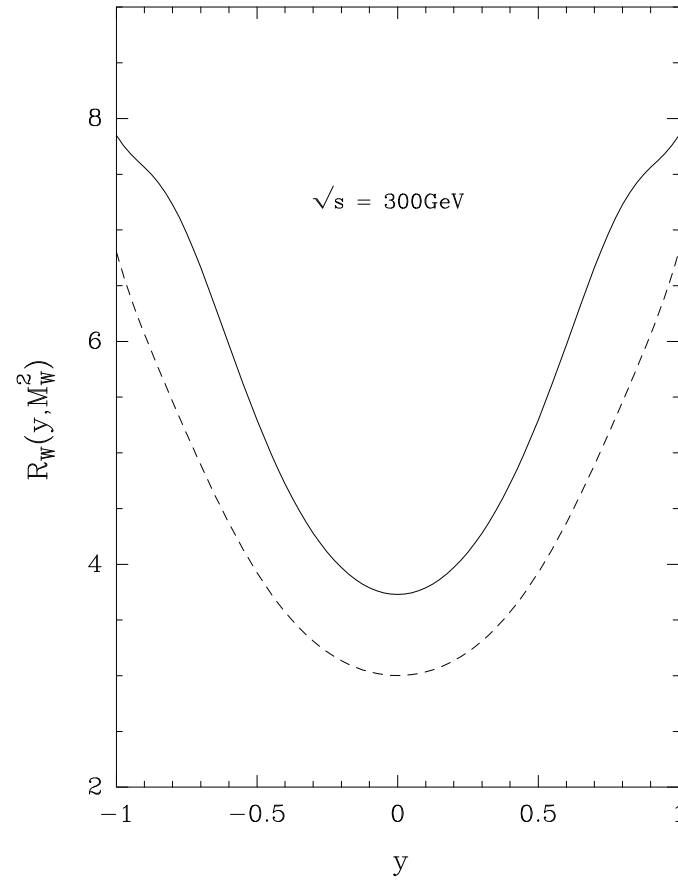
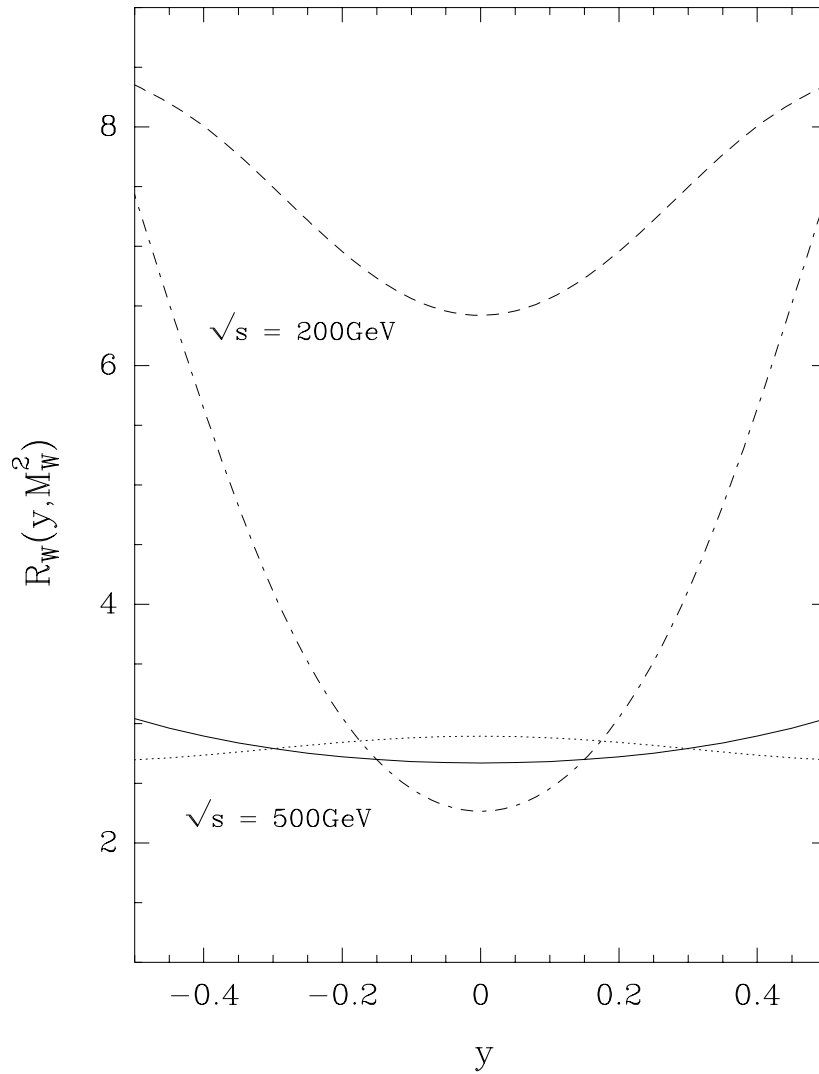
Light sea quarks asymmetry in $pp \rightarrow W^\pm$

- Consider $R_W(y) = (d\sigma^{W^+}/dy)/(d\sigma^{W^-}/dy)$ which reads in lowest order

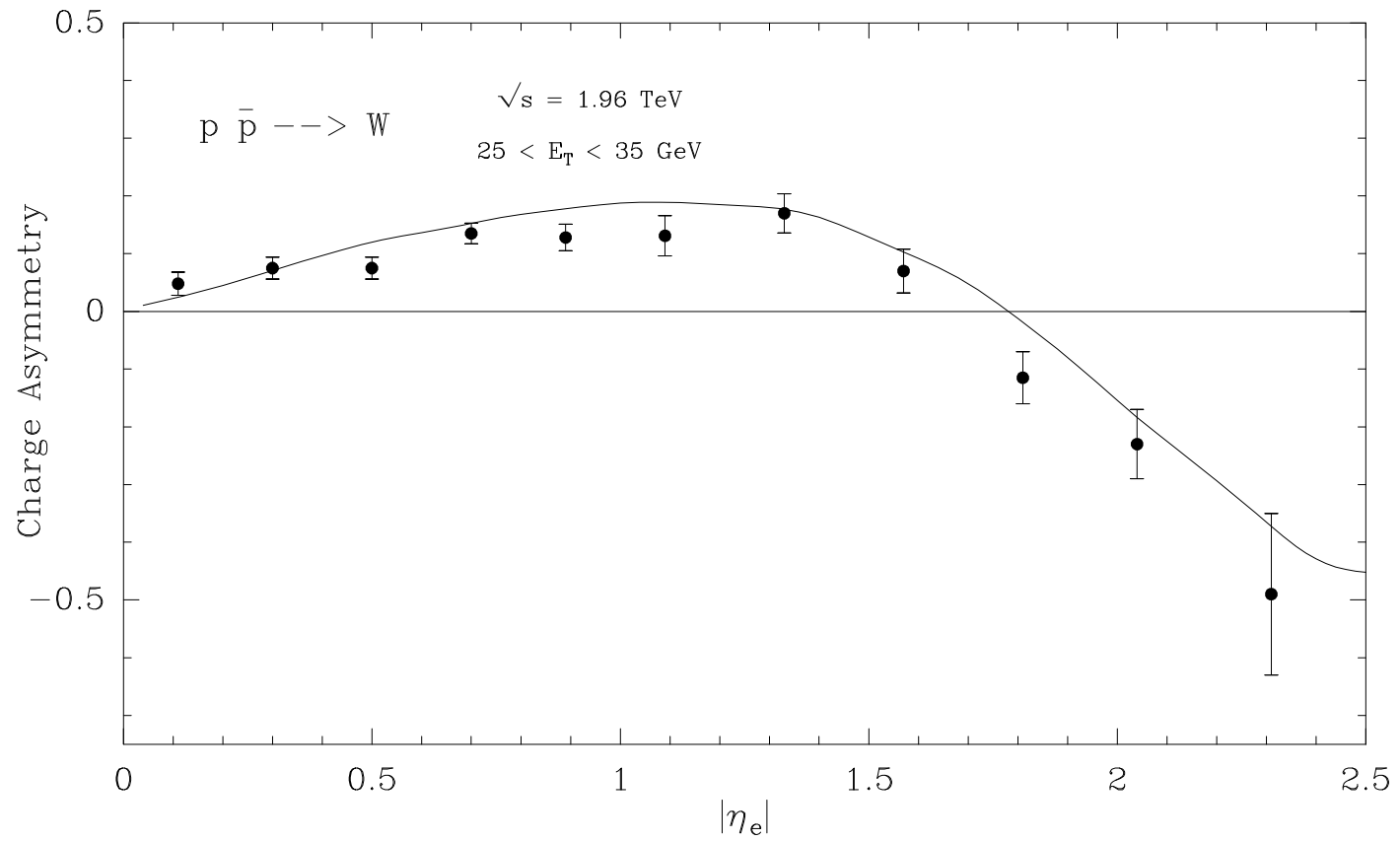
$$R_W(y, M_W^2) = \frac{u(x_a)\bar{d}(x_b) + \bar{d}(x_a)u(x_b)}{d(x_a)\bar{u}(x_b) + \bar{u}(x_a)d(x_b)}, \text{ where } x_a = \sqrt{\tau}e^y, \\ x_b = \sqrt{\tau}e^{-y} \text{ and } \tau = M_W^2/s.$$

- At $\sqrt{s} = 500\text{GeV}$ for $y = 0$, we have $x_a = x_b = 0.16$. So $R_W(0, M_W^2)$ probes the $\bar{d}(x)/\bar{u}(x)$ ratio at $x = 0.16$.
At $\sqrt{s} = 200\text{GeV}$ for $y = 0$, we have $x_a = x_b = 0.40$
Excellent test, but production rate is lower. (**Feasibility?**)
May be 300GeV , $x_a = x_b = 0.27$, is good enough, if can distinguish $R_W(0, M_W^2) \sim 4$ and 3 .

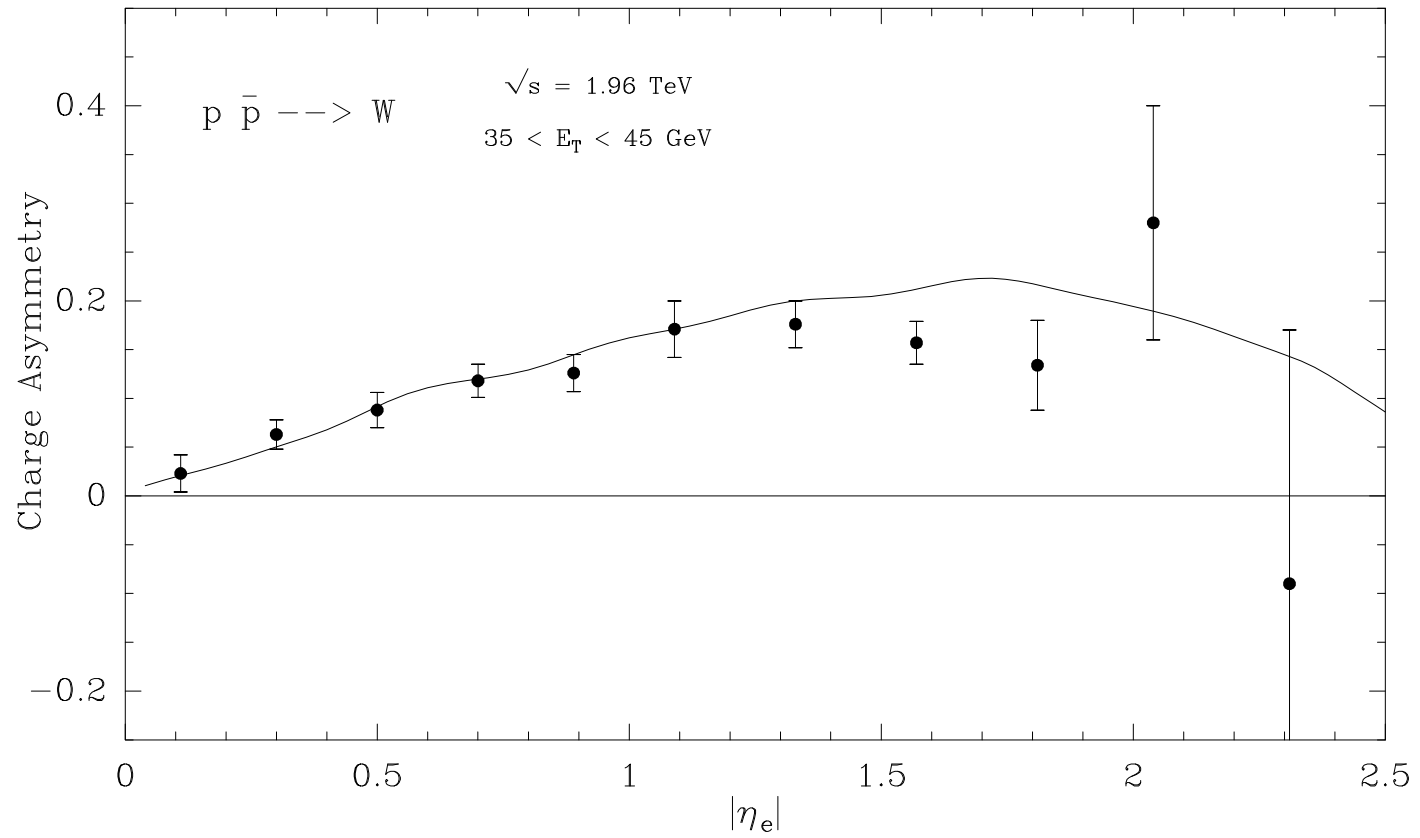
Decisive test at RHIC



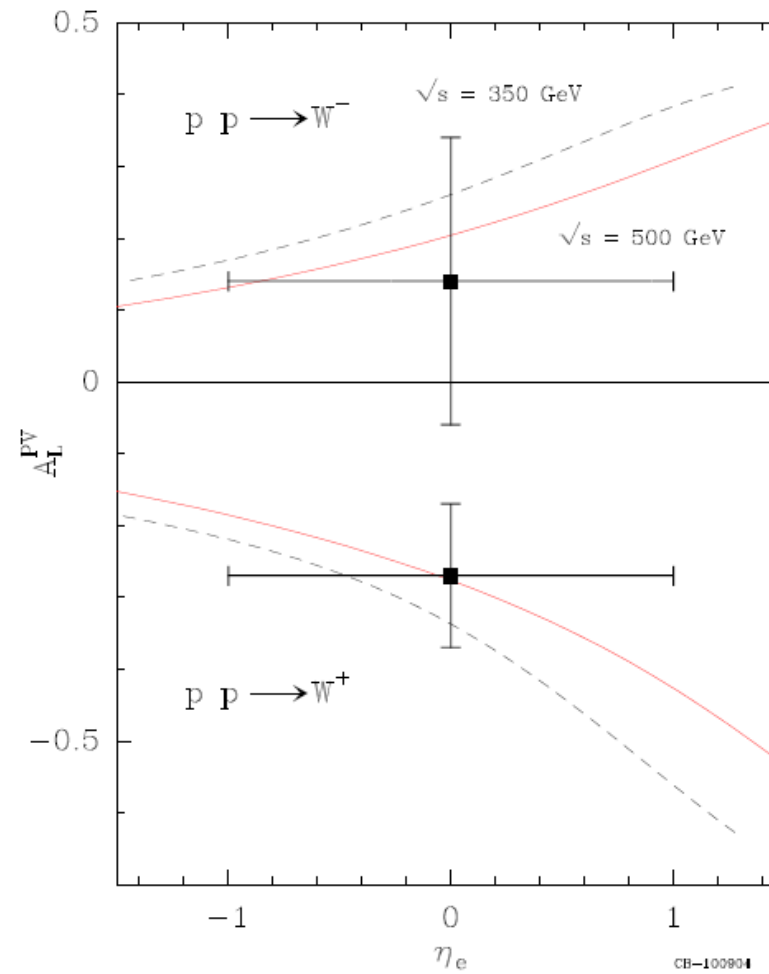
CDF data PRD71,051104(R) 2005(using RhicBos)



CDF data PRD71,051104(R) 2005(using RhicBos)



Parity-violating asymmetry in W^\pm production



Conclusions

- A new set of PDF is constructed in the framework of a statistical approach of the nucleon.
- All **unpolarized and polarized** distributions depend upon **nine** free parameters for light quarks and gluon, with some physical meaning.
- New tests against experimental (unpolarized and polarized) data on DIS, Semi-inclusive DIS and hadronic processes are very satisfactory.
- Good predictive power but some special features remain to be verified, specially **in the high x region**.

For practical use of our PDF see www.cpt.univ-mrs.fr/~bourelly/research/bbs-dir/bbs.html